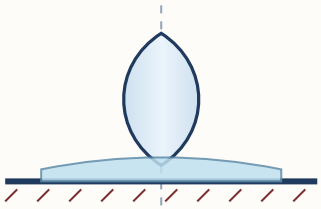
# PROJECT 03

## Determination of the Refractive Indices of Water and Transparent Oil

by the method of the liquid lens, using a plane mirror, an equiconvex lens and an adjustable object needle



$\mu = 1 + R/f_l$

**Name** _____

**Roll No.** _____

**School** _____

**Session** _____

Signature of Candidate : _____ Signature of Teacher : _____

# CERTIFICATE & INDEX

## Certificate

This is to certify that this record contains a true and complete account of the work performed by the candidate named on the cover in the Physics laboratory for the session indicated therein. The observations entered in the following pages were taken by the candidate with the apparatus set up under the supervision of the teacher-in-charge.

Date : _____

Place : _____

_____

Teacher-in-charge

Physics Laboratory

## Index


| Page | Content | Page | Content |
|---|---|---|---|
| 1 | Cover | 12 | Observations — spherometer |
| 2 | Certificate & index | 13 | Calculation — radius of curvature |
| 3 | Aim & introduction | 14 | Observations — water lens |
| 4 | Apparatus & material | 15 | Observations — oil lens |
| 5 | Theory I — focal length of lens | 16 | Calculations — μ and error |
| 6 | Theory II — the liquid lens | 17 | Result & discussion |
| 7 | Theory III — working formulae | 18 | Precautions |
| 8 | The spherometer | 19 | Sources of error |
| 9 | Procedure A — radius of curvature | 20 | Viva-voce |
| 10 | Procedure B — water lens | 21 | Conclusion & references |
| 11 | Procedure C — oil lens | | |


The theory of this project is the same as that of Experiment 07 of the Laboratory Manual — *To find the refractive index of a liquid by using a convex lens and a plane mirror.* The project extends the method to a second transparent liquid.

# AIM & INTRODUCTION

**AIM**

To find the refractive indices of (a) water and (b) transparent oil (kerosene) using a plane mirror, an equiconvex lens made of glass of known refractive index, and an adjustable object needle.

## Introduction

The refractive index of a transparent medium is one of its most characteristic optical constants. It is defined as the ratio of the speed of light in vacuum to the speed of light in that medium. Since the speed of light in a liquid is smaller than in air, the value is always greater than one; for clean water it is close to 4/3 and for kerosene it is close to 1.47. A measurement of refractive index therefore serves both as an identification of the liquid and as a sensitive test of its purity.

Ordinarily a liquid has no plane surface of its own that can be used in a lens formula. In this method the liquid is given a well-defined optical surface by pouring it into the thin gap between a convex lens and a plane mirror. The liquid then takes the shape of a **plano-concave lens**: one surface is the curved lower surface of the glass lens and the other is the plane mirror itself. The upper curved surface has the same radius of curvature as the glass lens, so a single measurement of that radius with a spherometer is enough.

The plane mirror is not merely a support. Light from the object needle passes through the lens, is reflected at the mirror and passes through the lens a second time. The light thus traverses the lens system twice, which doubles the optical effect and, more importantly, allows the focal position to be located by the **method of coincidence**: the object needle and its own inverted image are made to coincide, so no screen is required and the error of judging the sharpest image is avoided.

## What is measured in this project

1. **R** — the radius of curvature of the spherical surface of the convex lens, with a spherometer.
2. **f** — the focal length of the convex lens alone, with the plane mirror.
3. **f′** — the focal length of the convex lens + water-lens combination, and hence the focal length of the water lens.
4. **f″** — the corresponding focal length with kerosene oil in place of water.
5. The two refractive indices are then obtained from $\mu = 1 + R / f_{liquid}$.

**Scope of the result.** The refractive index so found is the refractive index of the liquid relative to air. Temperature and the wavelength of the light used are not varied in the experiment, so the value obtained is understood to refer to light from the source used at the temperature of the laboratory.

## APPARATUS & MATERIAL REQUIRED

The following items are arranged on a level working bench before the experiment is begun. Each is checked for cleanliness and for mechanical freedom of movement.

| S.No. | Item | Use in the experiment |
|---|---|---|
| 1 | Equiconvex lens | Forms the main converging lens; its lower surface shapes the liquid lens. Glass of known refractive index (≈ 1.5), focal length about 20 cm. |
| 2 | Plane mirror | Slightly larger than the aperture of the lens. Reflects the rays back through the lens and provides the plane surface of the liquid lens. |
| 3 | Optical needle | Adjustable object with a sharp tip; its tip and the tip of its image are made to coincide. |
| 4 | Retort stand with clamp | Holds the needle horizontally and allows fine vertical movement. |
| 5 | Clean water | First liquid under test; poured into the space between lens and mirror. |
| 6 | Transparent oil (kerosene) | Second liquid under test, used in the repetition of the procedure. |
| 7 | Dropper | Introduces the liquid without disturbing the position of the lens on the mirror. |
| 8 | Spherometer | Measures the sagitta of the lens surface, from which the radius of curvature is calculated. |
| 9 | Plumb line and half-metre scale | Set the needle vertically above the optical centre and measure the needle-to-lens distances. |

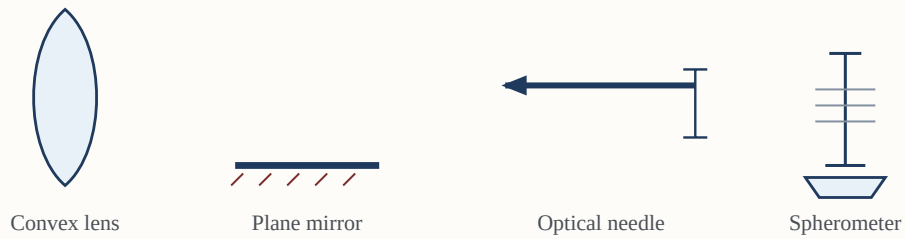



*Fig. 3 — Principal items of apparatus (schematic, not to scale).*

**Before starting:** the lens must be cleaned on both faces, the mirror must be clean and undistorted, and the tip of the needle must be free of bends. A dusty lens or a chipped needle tip is a common source of a diffuse, unmatchable image.

# THEORY I — FOCAL LENGTH OF THE CONVEX LENS

Place a convex lens on the reflecting surface of a plane mirror, with the mirror horizontal. Fix an object needle AB horizontally in the clamp of a retort stand above the lens, with its tip B on the vertical axis of the lens. When the needle is at the principal focus F of the lens, rays from B emerging from the lens are parallel to the axis. These rays fall normally on the plane mirror and retrace their path. After a second passage through the lens they converge at B itself, producing a real and inverted image A′B′ which coincides with the object.

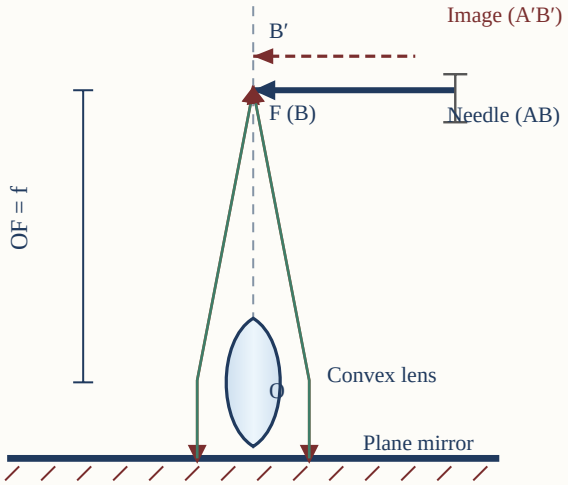



*Fig. 4 — Location of the principal focus of the convex lens using a plane mirror.*

## Working relation

Let $d_1$ and $d_2$ be the distances of the tip of the needle from the upper and lower surfaces of the lens when there is no parallax. The distance asked for is from the optical centre O, which lies between those two surfaces. Hence the mean is taken:

$$f = OF = \frac{d_1 + d_2}{2}$$

The condition of no parallax is tested by moving the eye slightly from side to side: the needle tip and its image must remain superimposed, with no relative shift. Only then is the distance read on the half-metre scale.

## THEORY II — THE LIQUID LENS

Now fill the space between the lens and the mirror with clean water, using a dropper, and allow the water to come to rest. The water occupies the shallow cavity bounded above by the convex lower surface of the glass lens and below by the plane mirror. It is therefore a **plano-concave lens of water**: thin at the centre and thick at the edges. Such a lens is diverging.

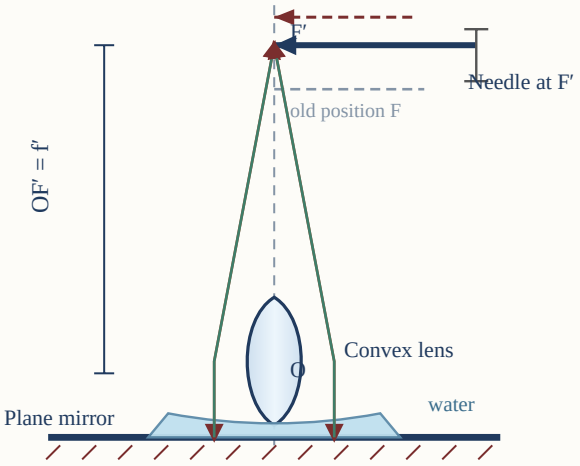



*Fig. 5 — The water lens formed between the convex lens and the mirror; the needle has been raised to F′.*

### Why the needle is moved upward

The water lens is diverging, so it reduces the total converging power of the combination. The focal length of the combination is therefore greater than f, and the focus moves to a point F′ farther from the lens. The needle must be raised until its tip again coincides with the tip of its image. With a liquid of larger refractive index the diverging action is stronger, so the new focus is still farther away; this is why the readings with kerosene are larger than those with water.

$$f' = OF' = \frac{d_3 + d_4}{2}$$

where $d_3$ and $d_4$ are the distances of the needle tip from the top and bottom surfaces of the lens in the presence of the liquid.

# THEORY III — WORKING FORMULAE

## 1. Focal length of the liquid lens

For two thin lenses in contact, the powers add. If f is the focal length of the convex lens, $f_l$ the signed focal length of the liquid lens and f′ that of the combination, then

$$\frac{1}{f'} = \frac{1}{f} + \frac{1}{f_l}$$

The liquid lens is diverging, so $f_l$ is negative. Writing $F_l = |f_l|$ for its positive magnitude,

$$F_l = \frac{f f'}{f' - f} \quad (\text{since } f' > f)$$

> The manual often writes the same relation as $f_w = ff'/(f - f')$, treating the focal length of the water lens as a *signed* negative quantity. The two statements are identical; in this record the positive magnitude $F_l$ is used so that the refractive index comes out greater than one.

## 2. Refractive index from the lens maker's formula

For a thin lens whose first surface has radius $R_1$ and the second $R_2$, immersed in air,

$$\frac{1}{F_l} = (\mu_l - 1)\left(\frac{1}{R_1} - \frac{1}{R_2}\right)$$

For the liquid lens the surface touching the plane mirror is plane, so $R_2 = \infty$ and $1/R_2 = 0$. The other surface has the radius R of the lower surface of the glass lens; with the sign convention of the experiment its magnitude is taken. Hence

$$\frac{1}{F_l} = (\mu_l - 1)\frac{1}{R} \quad \Rightarrow \quad \boldsymbol{\mu_l = 1 + \frac{R}{F_l}}$$

Thus only two measured quantities enter the result: the radius of curvature R and the focal length $F_l$ of the liquid lens.

## 3. Cross-check using the known glass refractive index

The glass lens is equiconvex, so both its surfaces have the same radius R. Its focal length satisfies

$$\frac{1}{f} = (\mu_g - 1)\frac{2}{R} \quad \Rightarrow \quad R = 2f(\mu_g - 1)$$

With $\mu_g = 1.50$ and $f \approx 20$ cm this gives $R \approx 20$ cm, which agrees with the value obtained independently with the spherometer. Agreement between the two values is a useful check that the lens has been identified correctly and that the spherometer reading has been taken properly.

## 4. Summary of symbols

| Symbol | Meaning | Value used |
|---|---|---|
| R | Radius of curvature of the lens surface | 20.04 cm |
| f | Focal length of convex lens alone | ≈ 20.0 cm |
| f′, f″ | Focal length of combination with water, with oil | ≈ 29.9, 37.7 cm |
| $F_w$, $F_o$ | Focal length of the water and oil lenses | ≈ 60.2, 42.5 cm |
| $\mu_w$, $\mu_o$ | Refractive indices to be determined | ≈ 1.33, 1.47 |

# THE SPHEROMETER

A spherometer measures the radius of curvature of a spherical surface. It consists of a rigid frame with three legs forming an equilateral triangle of side l and a fine central screw carrying a circular scale. The screw is moved until its tip and the three legs all just touch the surface to be measured.

## Pitch and least count

The **pitch** is the linear distance advanced by the screw in one complete rotation of the circular disc. The **least count** is the smallest distance that can be read:

$$LC = \frac{pitch}{number\ of\ divisions\ on\ circular\ scale}$$

For the instrument used here, pitch = 1.00 mm and the circular scale has 100 divisions, so LC = 0.01 mm = 0.001 cm.

## Sagitta and radius of curvature

The difference between the readings of the screw on the spherical (curved) surface and on a plane glass sheet is the **sagitta h** — the height of the spherical cap above the plane through the three legs. From the geometry of the circle, the distance from the centroid of the equilateral triangle to a leg is r = l/√3, and

$$(R - h)^2 + r^2 = R^2 \Rightarrow r^2 = 2Rh - h^2 \Rightarrow \mathbf{R = \frac{l^2}{6h} + \frac{h}{2}}$$

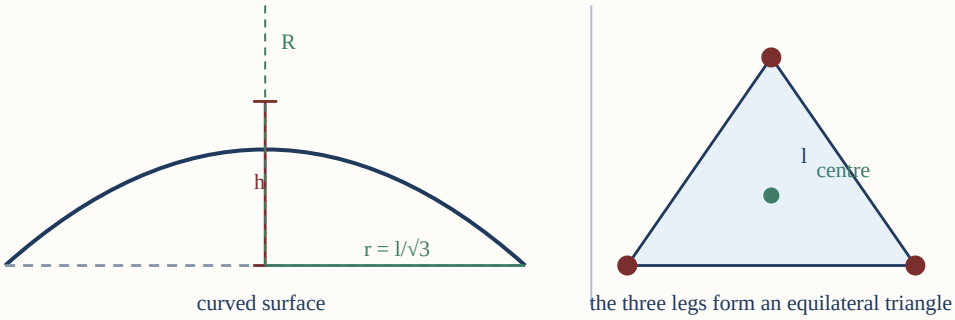



*Fig. 6 — Sagitta h of a spherical surface (left) and the triangular arrangement of the spherometer legs (right).*

**Reading the instrument.** Always turn the screw in one direction only while approaching the surface; reversing the motion introduces backlash error. When the screw tip just touches a reflecting surface, its image in the surface coincides with the tip itself — this is the most delicate way of judging contact.

# PROCEDURE A — RADIUS OF CURVATURE

### Initial observations of the spherometer

1. Take the spherometer and examine it carefully: its three legs and the vertical scale must not be shaky, and the central screw must not be loose.
2. Note the smallest division on the vertical (pitch) scale of the instrument.
3. Note the total number of divisions N on the circular scale.
4. Calculate the least count of the spherometer from LC = pitch / N.

### To find the pitch of the screw

5. Place the spherometer on a plane glass slab and turn the central screw until its tip just touches the slab.
6. While turning the screw, count the number of complete rotations p required to advance it through the known distance q measured on the vertical scale. Then pitch = q / p.

### Setting up the three legs

7. Place the spherometer on a sheet of paper and press it lightly so that the three leg tips leave clear impressions.
8. Join the three impressions to form the equilateral triangle ABC. Measure AB, BC and CA and calculate l = (AB + BC + CA)/3.

### Plane surface reading

9. Place the spherometer on the plane glass slab and bring the central screw down gently until it touches the slab. The tip and its image in the glass then appear coincident.
10. Read the main scale: m is the number of complete main-scale divisions below the edge of the circular scale.
11. Read the circular scale: c is the division that coincides with the edge of the main scale. The total reading is m × (LC of main scale) + c × (LC of spherometer).

### Curved surface reading

12. Place the spherometer on the spherical surface of the convex lens with its three legs resting on the glass, as in Fig. 6.
13. Move the central screw until its tip just touches the curved surface; the touch must be gentle, since a glass lens is easily scratched.
14. Note the main scale and circular scale readings. Observe whether the zero of the circular scale has moved up or down relative to the main scale.
15. The difference between the curved-surface and plane-surface readings, with the proper sign, gives the sagitta h.
16. Repeat the readings at least four times and calculate the mean h.

**Important.** Turn the screw in one direction only while approaching the surface; if it is reversed, the screw threads have play and a backlash error is introduced. The mean of the corrected h values is used in $R = l^2/6h + h/2$.

# PROCEDURE B — CONVEX LENS AND WATER LENS

## Setting up the apparatus

1. Estimate the rough focal length of the convex lens, if it is not already known, by forming a sharp image of a distant object on a screen and measuring the image distance.
2. Place a plane mirror on a horizontal base with its reflecting surface facing upward.
3. Place the convex lens of comparatively small aperture on the mirror, at its centre.
4. Fix a small optical needle AB horizontally in the clamp of the retort stand above the lens. Using a plumb line and a spirit level, adjust the needle so that its tip lies vertically above the optical centre O of the lens.

## Focal length of the convex lens alone

5. Look into the lens from above and move the clamped needle slowly upward or downward until the tip of the needle and the tip of its inverted image are at the same level.
6. Remove the parallax completely by moving the eye slightly from side to side: the two tips must move together and remain coincident.
7. Note $d_1$, the distance of the needle tip from the upper surface of the lens, and $d_2$, the distance from the lower surface. Calculate $f = (d_1 + d_2)/2$.
8. Repeat the setting three more times and enter all readings in the table.

## Focal length with water — the water lens

9. Using a dropper, fill the open space between the convex lens and the plane mirror with clean water. Wait for a few minutes so that the water becomes perfectly stationary and the lens does not shift on the mirror.
10. The water now forms a plano-concave lens in contact with the convex lens. Its diverging action pushes the focus upward. Raise the clamp slowly until the parallax between the needle tip and its inverted image is removed once again.
11. Measure $d_3$ and $d_4$, the distances of the needle tip from the top and bottom surfaces of the lens, and calculate $f' = (d_3 + d_4)/2$.
12. Repeat three more times. Compute the focal length of the water lens for each set from $F_w = f\,f'/(f' - f)$.

**Why the average of two surfaces?** The optical centre of a lens is not directly accessible to a scale. It lies between the two surfaces, so the mean of the distances read to the upper and lower surfaces gives the distance to the optical centre with the thickness error largely cancelled.

# PROCEDURE C — TRANSPARENT OIL & PRECAUTIONS IN METHOD

## Repeating the experiment with oil

1. Remove the water carefully with a dropper or absorbent paper. Clean and dry the convex lens and the plane mirror so that no trace of water remains.
2. Place the lens back in exactly the same position on the mirror. Check once more that the needle tip is vertically above the optical centre.
3. Introduce transparent kerosene oil into the space between the lens and the mirror with a clean dropper. Take care not to displace the lens and avoid trapping air bubbles in the thin liquid layer.
4. Wait until the oil layer is still. The greater refractive index of oil makes the liquid lens more strongly diverging, so the needle has to be raised still higher.
5. Remove parallax as before and note $d_3'$ and $d_4'$ for four independent settings. Calculate $f'' = (d_3' + d_4')/2$.
6. Compute the focal length of the oil lens for each set from $F_o = f\,f''/(f'' - f)$, and then $\mu_o = 1 + R/F_o$.

## Points to be observed during the experiment

7. The image formed by the combination is inverted and of the same size as the needle; it lies at the level of the needle when parallax is removed.
8. The distance $f''$ with oil is larger than $f'$ with water, which in turn is larger than $f$ for the lens alone. This order is a direct check that the experiment has been performed correctly.
9. The scale readings are taken with the eye at the level of the needle tip to avoid oblique viewing.
10. After the experiment the oil and water are removed and the lens and mirror are cleaned and stored dry.

*Expected order of measured distances*

| Medium between lens and mirror | Nature of liquid lens | Focal length of combination |
|---|---|---|
| Air (no liquid) | None | $f \approx 20.0$ cm (smallest) |
| Water, $\mu \approx 1.33$ | Plano-concave, weakly diverging | $f' \approx 29.9$ cm |
| Kerosene, $\mu \approx 1.47$ | Plano-concave, more diverging | $f'' \approx 37.7$ cm (largest) |

**Handling transparent oil.** Kerosene spreads easily and leaves a film that changes the wetting of the lens surface. Use only a few drops, fill slowly from one edge so that air can escape, and do not allow the oil to reach the mirror edge or the stand.

# OBSERVATIONS — SPHEROMETER

## Constants of the instrument

| | |
|---|---|
| Smallest division on the vertical (pitch) scale | 1.00 mm |
| Distance q moved in p = 1 complete rotation | 1.00 mm |
| Pitch of the screw, q / p | 1.00 mm |
| Total number of divisions N on the circular scale | 100 |
| Least count of the spherometer, pitch / N | 0.01 mm = 0.001 cm |

## Leg separation

Impressions of the three legs: AB = 3.02 cm, BC = 2.98 cm, CA = 3.00 cm.

$$l = \frac{AB + BC + CA}{3} = \frac{3.02 + 2.98 + 3.00}{3} = 3.00\ cm$$

## Table 1 — Sagitta of the convex lens surface

*LC of spherometer = 0.001 cm; MSR = main scale reading; CSR = circular scale reading.*

| S.N. | Curved surface reading | | | | Plane surface reading | | | | $h = h_A - h_B$ (cm) |
|---|---|---|---|---|---|---|---|---|---|
| | MSR x (cm) | CSD y | CSR z = y×LC (cm) | $h_A$ = x+z (cm) | MSR x′ (cm) | CSD y′ | CSR z′ = y′×LC (cm) | $h_B$ = x′+z′ (cm) | |
| 1 | 0.000 | 74 | 0.074 | 0.074 | 0.000 | 0 | 0.000 | 0.000 | 0.074 |
| 2 | 0.000 | 75 | 0.075 | 0.075 | 0.000 | 0 | 0.000 | 0.000 | 0.075 |
| 3 | 0.000 | 76 | 0.076 | 0.076 | 0.000 | 0 | 0.000 | 0.000 | 0.076 |
| 4 | 0.000 | 75 | 0.075 | 0.075 | 0.000 | 0 | 0.000 | 0.000 | 0.075 |
| **Mean sagitta** | | | | | | | | | **0.075** |

The screw was advanced in the same direction for every setting; the zero of the circular scale moved downward on the curved surface, so the corrected sagitta is taken as positive. Mean h = (0.074 + 0.075 + 0.076 + 0.075)/4 = **0.075 cm**.

## CALCULATION — RADIUS OF CURVATURE

From Table 1 the mean sagitta of the spherical surface of the convex lens is h = 0.075 cm, and the mean separation of the spherometer legs is l = 3.00 cm. The radius of curvature is therefore

$$R = \frac{l^2}{6h} + \frac{h}{2}$$

$$= \frac{(3.00)^2}{6 \times 0.075} + \frac{0.075}{2}$$

$$= \frac{9.00}{0.450} + 0.0375 = 20.00 + 0.0375$$

$$\mathbf{R = 20.04\ cm}$$

### Individual values from the four settings

| Setting | h (cm) | R = l²/6h + h/2 (cm) |
|---|---|---|
| 1 | 0.074 | 20.31 |
| 2 | 0.075 | 20.04 |
| 3 | 0.076 | 19.77 |
| 4 | 0.075 | 20.04 |
| **Mean radius of curvature** | | **20.04 cm** |

### Independent check from the focal length of the lens

The focal length of the equiconvex glass lens measured in Section B is about 20.0 cm and the refractive index of its glass is known to be $\mu_g = 1.50$. For an equiconvex lens,

$$R = 2f(\mu_g - 1) = 2 \times 20.0 \times (1.50 - 1) = 20.0\ cm$$

The value obtained from the spherometer, 20.04 cm, agrees with this to within 0.2%. The agreement confirms that the lens surface used for the liquid lens has indeed the radius R inserted in the working formula.

**Uncertainty in R.** The least count of the vertical scale is 0.001 cm. Taking a conservative uncertainty of ±0.05 cm from the scatter of the four settings, the fractional uncertainty ΔR/R is about 0.25%, small compared with the uncertainty in the focal-length measurements.

## OBSERVATIONS — FOCAL LENGTH WITH WATER

Distances are measured from the tip of the needle. f is the focal length of the convex lens alone; f′ is that of the convex lens + water lens; $F_w = f\,f'/(f' - f)$ is the focal length of the water lens alone.

*Table 2 — Determination of μ of water. Mean R = 20.04 cm; least count of scale = 0.1 cm.*

| S.N. | Without water (OF = f) | | | With water (OF′ = f′) | | | $F_w = ff'/f'-f$ (cm) | $\Delta F_w$ (cm) | $\mu_w = 1 + R/F_w$ | $\Delta\mu_w$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $d_1$ (cm) | $d_2$ (cm) | $f = (d_1+d_2)/2$ (cm) | $d_3$ (cm) | $d_4$ (cm) | $f' = (d_3+d_4)/2$ (cm) | | | | |
| 1 | 19.8 | 20.2 | 20.00 | 29.6 | 30.2 | 29.90 | 60.40 | 1.3 | 1.332 | 0.03 |
| 2 | 19.9 | 20.1 | 20.00 | 29.7 | 30.1 | 29.90 | 60.40 | 1.3 | 1.332 | 0.03 |
| 3 | 20.0 | 20.2 | 20.10 | 30.0 | 30.4 | 30.20 | 60.10 | 1.3 | 1.333 | 0.03 |
| 4 | 19.7 | 20.1 | 19.90 | 29.5 | 30.1 | 29.80 | 59.90 | 1.4 | 1.335 | 0.03 |
| **Mean values** | | | **20.00** | | | **29.95** | **60.20** | **1.3** | **1.333** | **0.03** |

### Sample calculation for setting 1

$$f = \frac{19.8 + 20.2}{2} = 20.00\ \text{cm} \qquad f' = \frac{29.6 + 30.2}{2} = 29.90\ \text{cm}$$

$$F_w = \frac{20.00 \times 29.90}{29.90 - 20.00} = \frac{598.0}{9.90} = 60.40\ \text{cm}$$

$$\mu_w = 1 + \frac{20.04}{60.40} = 1 + 0.3318 = \mathbf{1.332}$$

The mean focal length of the water lens is $F_w$ = 60.20 cm, considerably larger than the focal length of the glass lens; this is expected because a plano-concave water lens is weakly diverging.

## OBSERVATIONS — FOCAL LENGTH WITH OIL

The same procedure is repeated with transparent kerosene oil. Here f″ is the focal length of the combination and $F_o$ = f f″/(f″ − f) is the focal length of the oil lens.

*Table 3 — Determination of μ of kerosene oil. Mean R = 20.04 cm.*

| S.N. | Without oil (OF = f) | | | With oil (OF″ = f″) | | | $F_o = ff''/f''-f$ (cm) | $\Delta F_o$ (cm) | $\mu_o = 1 + R/F_o$ | $\Delta\mu_o$ |
|---|---|---|---|---|---|---|---|---|---|---|
| | $d_1$ (cm) | $d_2$ (cm) | $f = (d_1+d_2)/2$ (cm) | $d_3'$ (cm) | $d_4'$ (cm) | $f'' = (d_3'+d_4')/2$ (cm) | | | | |
| 1 | 19.8 | 20.2 | 20.00 | 37.4 | 38.0 | 37.70 | 42.60 | 0.58 | 1.470 | 0.02 |
| 2 | 19.9 | 20.1 | 20.00 | 37.5 | 38.1 | 37.80 | 42.47 | 0.58 | 1.472 | 0.02 |
| 3 | 20.0 | 20.2 | 20.10 | 37.8 | 38.4 | 38.10 | 42.55 | 0.59 | 1.471 | 0.02 |
| 4 | 19.7 | 20.1 | 19.90 | 37.2 | 37.8 | 37.50 | 42.40 | 0.58 | 1.473 | 0.02 |
| **Mean values** | | | **20.00** | | | **37.78** | **42.51** | **0.58** | **1.471** | **0.02** |

### Sample calculation for setting 1

$$f'' = \frac{37.4 + 38.0}{2} = 37.70 \text{ cm}$$

$$F_o = \frac{20.00 \times 37.70}{37.70 - 20.00} = \frac{754.0}{17.70} = 42.60 \text{ cm}$$

$$\mu_o = 1 + \frac{20.04}{42.60} = 1 + 0.4704 = \mathbf{1.470}$$

**Comparison.** The focal length of the oil lens (≈ 42.5 cm) is smaller than that of the water lens (≈ 60.2 cm). A liquid of larger refractive index produces, for the same surface curvature, a more strongly diverging plano-concave lens and therefore a smaller focal length. This is why the combination with oil has the largest needle distance f″ ≈ 37.8 cm.

# CALCULATIONS — REFRACTIVE INDICES & ERROR

## 1. Refractive index of water

$$\text{Mean } F_w = \frac{60.40 + 60.40 + 60.10 + 59.90}{4} = 60.20 \text{ cm}$$

$$\mu_w = 1 + \frac{R}{F_w} = 1 + \frac{20.04}{60.20} = \mathbf{1.333}$$

## 2. Refractive index of kerosene oil

$$\text{Mean } F_o = \frac{42.60 + 42.47 + 42.55 + 42.40}{4} = 42.51 \text{ cm}$$

$$\mu_o = 1 + \frac{R}{F_o} = 1 + \frac{20.04}{42.51} = \mathbf{1.471}$$

## 3. Propagation of error

Since μ = 1 + R/F, where F = ff′/(f′ − f), the error in the focal length of the liquid lens is

$$\Delta F = F^2 \frac{\Delta f}{f^2} + F^2 \frac{\Delta f'}{f'^2}$$

and the fractional error in the refractive index is

$$\Delta \mu = \mu \frac{\Delta R}{R} + \mu \frac{\Delta F}{F}$$

Taking Δf = Δf′ = 0.1 cm (least count of the half-metre scale) and ΔR = 0.05 cm:

| Quantity | Water | Kerosene oil |
|---|---|---|
| F (cm) | 60.20 | 42.51 |
| ΔF = F²(Δf/f² + Δf′/f′²) (cm) | 1.31 | 0.58 |
| ΔF / F | 0.0218 | 0.0136 |
| ΔR / R | 0.0025 | 0.0025 |
| Δμ = μ(ΔR/R + ΔF/F) | 0.032 → 0.03 | 0.024 → 0.02 |

## Final results

$$\mu_{\text{water}} = 1.33 \pm 0.03 \qquad \mu_{\text{kerosene}} = 1.47 \pm 0.02$$

The maximum of the four individual values of Δμ is reported with each result, as prescribed in the manual. The uncertainty is dominated by the parallax setting of the needle, not by the spherometer reading.

# RESULT & DISCUSSION

## Result

The refractive indices of the two liquids, measured relative to air by the liquid-lens method, are:

(a) Refractive index of clean water, $\mu_w$ = **1.33 ± 0.03**

(b) Refractive index of transparent kerosene oil, $\mu_o$ = **1.47 ± 0.02**

## Comparison with standard values

| Liquid | Standard value | Measured value | Percentage deviation |
|---|---|---|---|
| Water | 1.333 | 1.333 | ≈ 0.0% |
| Kerosene oil | ≈ 1.47 | 1.471 | ≈ 0.1% |

Both measured values lie within the estimated experimental error of the accepted values, so the experiment may be regarded as satisfactory.

## Discussion

1. The three focal lengths satisfy f < f′ < f″. This is the expected order: adding a diverging liquid lens reduces the converging power of the system, and kerosene, having the larger refractive index, reduces it more than water does.
2. Equivalently, the focal length of the liquid lens itself is smaller for oil (≈ 42.5 cm) than for water (≈ 60.2 cm), in accordance with $F_l = R/(\mu_l - 1)$.
3. The radius of curvature obtained with the spherometer (20.04 cm) agrees with the value calculated from the known refractive index of the glass lens and its focal length (20.0 cm). This cross-check shows that no systematic error has entered the spherometer measurement.
4. The main limitation is the judgment of coincidence between the needle tip and its image. A small residual parallax shifts f′ by a few tenths of a centimetre and, because $F_l$ depends on the difference f′ − f, the effect is magnified in the final refractive index.
5. The thin-lens approximation is used throughout; the finite thickness of the lens is partly compensated by averaging the distances measured from its two surfaces. Temperature changes during the experiment would alter the refractive index slightly and were kept small by performing both sets of readings in the same session.

**Inference.** The liquid-lens method is a convenient way of finding the refractive index of a liquid without a prism or a spectrometer, requiring only a lens, a mirror, a needle and a scale. Its accuracy is limited by the parallax setting but is adequate for laboratory work.

## PRECAUTIONS

### A. In determining the radius of curvature of the convex lens

1. The central screw of the spherometer must be moved in one direction only while approaching a surface; reversing it introduces backlash error.
2. The three legs of the spherometer must all touch the spherical surface; if one leg lifts, the measured sagitta is wrong.
3. The screw at the centre must touch the surface gently, just enough for the tip and its image to coincide. Excessive pressure scratches the glass and depresses the setting.
4. The glass slab used for the plane-surface reading and the lens surface must both be clean and free of dust.
5. The impressions of the three legs on paper should be sharp, and the triangle should be measured with a fine scale.
6. At least four settings should be taken and the mean sagitta used in the calculation.

### B. In determining the refractive index of water

7. The convex lens must be so adjusted on the mirror that its principal axis is vertical with respect to the mirror surface.
8. A thin lens of comparatively small aperture should be used, so that the distance measured from the surface of the lens is nearly equal to the distance from the optical centre.
9. The needle tip must lie vertically above the optical centre of the lens; use a plumb line and a spirit level for the initial setting.
10. Water should be poured slowly into the open space with a dropper, so that the position of the lens on the mirror does not change.
11. Wait for a few minutes after filling, until the water becomes stationary, before taking readings.
12. It is better to measure the distance from both the upper and lower surfaces of the lens and take the mean, for f as well as for f′.
13. Parallax must be removed completely: the eye is moved slightly sideways and the two tips must remain coincident.

### C. In repeating the experiment with oil

14. The lens and mirror must be thoroughly dried before kerosene is introduced; mixing the two liquids would give an intermediate and meaningless value.
15. Use a clean, separate dropper for the oil and avoid air bubbles in the liquid layer.
16. Do not disturb the position of the lens while filling; replace it in the same position if it has been lifted.
17. Clean the apparatus after the experiment and store the lens and mirror dry.

> The precautions are the same as those prescribed for Experiment 07 of the laboratory manual, with the additional care needed because two different liquids are used in succession.

# SOURCES OF ERROR & ERROR ANALYSIS

## A. In determining the radius of curvature

1. Graduations on the pitch and circular scales may not be perfectly uniform.
2. Instrumental and human errors are possible in reading the scales and in judging contact of the screw tip.
3. Parallax error may arise while reading the pitch scale corresponding to the level of the circular scale.
4. Backlash error occurs if the screw is turned back while taking a reading.
5. The legs may rest on dust particles, raising the spherometer and changing the sagitta.

## B. In determining the refractive index of the liquids

6. The image is slightly diffuse, so the exact position of no parallax is difficult to locate.
7. The radius of curvature of the plano-concave water lens may not be exactly equal to that of the glass lens if the lens is not perfectly equiconvex.
8. The needle fixed in the clamp may not be exactly horizontal or may be slightly bent at the tip.
9. The lens has finite thickness, whereas the lens maker's formula used assumes a thin lens.
10. The liquid layer may contain a small air bubble, or the lens may move slightly while the liquid is being poured.
11. A change in temperature between the two sets of readings changes the refractive index of the liquids.

## Error analysis

The refractive index is calculated from $\mu = 1 + R/F$, with $F = ff'/(f' - f)$. Differentiating,

$$\Delta\mu = \mu\,\frac{\Delta R}{R} + \mu\,\frac{\Delta F}{F}, \quad \text{where} \quad \Delta F = F^2\,\frac{\Delta f}{f^2} + F^2\,\frac{\Delta f'}{f'^2}$$

Here $\Delta f$, $\Delta f'$ and $\Delta R$ are the least counts of the scales with which the corresponding quantities are measured, and F is the mean focal length of the liquid lens. The largest of the four values obtained for $\Delta\mu$ is reported as the experimental error.

| Source of uncertainty | Magnitude | Effect on Δμ |
|---|---|---|
| Spherometer least count (ΔR) | 0.001 cm; taken as 0.05 cm | Small (≈ 0.25%) |
| Half-metre scale (Δf, Δf′) | 0.1 cm | Dominant (≈ 2%) |
| Parallax in coincidence | Few mm | Dominant and unavoidable |

Reported errors: $\mu_{water} = 1.33 \pm 0.03$ and $\mu_{oil} = 1.47 \pm 0.02$. The errors are dominated by the focal-length readings rather than by the measurement of the radius of curvature.

## VIVA-VOCE

**Q1. Can we measure the radii of curvature of both the surfaces of a plano-concave lens?**
**Ans.** No. A plano-concave lens has one concave surface and one plane surface. The radius of curvature of the plane surface is infinite and cannot be determined by a spherometer; only the curved spherical surface can be measured.

**Q2. For what purpose do we use a spherometer?**
**Ans.** A spherometer is used to measure the radii of curvature of spherical surfaces such as a concave mirror or a convex lens, by measuring the sagitta of the surface.

**Q3. In this experiment, at which condition is the parallax removed completely?**
**Ans.** Parallax is removed when the sharp tip of the needle and the tip of its inverted image are exactly at the same point, so that they appear as one even when the eye is moved slightly sideways.

**Q4. Find the refractive index of glass with respect to water ($\mu_{gw}$).**
**Ans.** $\mu_w = 4/3$ and $\mu_g = 3/2$, so $\mu_{gw} = \mu_g/\mu_w = (3/2)/(4/3) = 9/8 = 1.125$.

**Q5. On which factors does the refractive index of a material depend?**
**Ans.** It depends on (i) the wavelength of light used (colour) and (ii) the temperature of the medium. It also depends on the nature of the two media between which light travels.

**Q6. Write the values of refractive index of water and glass with respect to air.**
**Ans.** Water: $\mu_w = 4/3 \approx 1.33$. Glass: $\mu_g = 3/2 = 1.50$.

**Q7. Define refractive index.**
**Ans.** The absolute refractive index of a medium is the ratio of the speed of light in vacuum to the speed of light in that medium: $\mu = c/v$.

**Q8. Why should we move the needle upward after filling water between the lens and the mirror?**
**Ans.** The water and the convex lens act as a combined lens of lower converging power because the water lens is diverging. The image is therefore formed farther from the lens, so the needle is raised to bring its tip into coincidence with the image at the new position.

**Q9. What is the role of the plane mirror in this experiment?**
**Ans.** It sends the rays back through the lens, so the light passes through the lens twice. The object and its image then lie at the same place, which makes it possible to locate the focus by the method of coincidence without using a screen.

**Q10. Why is the liquid lens formed in this arrangement a plano-concave lens?**
**Ans.** The liquid is thinnest at the centre, where the lens surface is closest to the mirror, and thicker toward the edges. Its lower surface is the plane mirror and its upper surface follows the convex lower face of the glass lens, giving a plano-concave shape.

**Q11. Why is the mean of the distances from the upper and lower lens surfaces taken?**
**Ans.** The focal length must be measured from the optical centre of the lens, which is not directly accessible. Its position lies between the two surfaces, so the average of the two measured distances gives the required distance to a good approximation.

**Q12. Why is the focal length of the oil lens smaller than that of the water lens?**
**Ans.** From $F_l = R/(\mu_l - 1)$, a larger refractive index gives a smaller focal length. Kerosene ($\mu \approx 1.47$) is optically denser than water ($\mu \approx 1.33$), so its plano-concave lens is more strongly diverging and has the shorter focal length.

# CONCLUSION & REFERENCES

## Conclusion

The refractive indices of clean water and transparent kerosene oil have been determined by forming a plano-concave liquid lens between a convex lens and a plane mirror and measuring its focal length by the method of no parallax. The values obtained, 1.33 for water and 1.47 for kerosene, agree with the accepted values within the limits of experimental error. The experiment also illustrates the addition of powers of thin lenses in contact and the use of the lens maker's formula for a liquid lens.

## Applications of the principle

- Identification of an unknown transparent liquid and checking its purity from a refractive-index measurement.
- Determination of the concentration of solutions, for example sugar content in syrup, from the change in refractive index.
- Design of optical systems in which a liquid lens is combined with a glass lens, as in variable-focus liquid lenses.
- Use of immersion oils of matched refractive index in high-resolution microscopy.

## Precautions taken in recording

All observations have been entered at the time of the experiment. Distances are recorded in centimetres to the precision permitted by the scale, the least count of the spherometer is stated explicitly, and the mean of four settings is used for every calculated quantity. Sample numerical readings should be replaced by the candidate's own observations before submission.

## Formulae used — at a glance

| Quantity | Relation used |
|---|---|
| Least count of spherometer | LC = pitch / number of circular-scale divisions |
| Radius of curvature of lens | $R = l^2/(6h) + h/2$ |
| Focal length of convex lens | $f = (d_1 + d_2)/2$ |
| Focal length of combination with liquid | $f' = (d_3 + d_4)/2$ (f″ for oil) |
| Focal length of the liquid lens | $F_l = f\,f'/(f' - f)$ |
| Refractive index of the liquid | $\mu_l = 1 + R/F_l$ |
| Experimental error | $\Delta\mu = \mu(\Delta R/R) + \mu(\Delta F/F)$ |

## References

1. *Laboratory Manual — Physics, Class XII*, Experiment 07: To find the refractive index of a liquid by using a convex lens and a plane mirror; and Project 03: Determination of refractive indices of water and oil.
2. *Physics, Class XII, Part II*, Chapter 9 — Ray Optics and Optical Instruments: refraction at spherical surfaces, lens maker's formula, and combination of thin lenses in contact.

Candidate

Teacher-in-charge